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G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
IV	PART-III	CORE-11	P23MA411	FUNCTIONAL ANALYSIS

Date & Session : 24.04.2025/FN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer ALL Questions.
CO1	K1	1.	A continuous linear transformation is also named as ________________ linear transformation. a) continuous b) bounded c) convex d) concave
CO1	K2	2.	Can you "tell" what a Banach space is? a) complete normed linear space b) convergent linear space c) A vector space with no norm d) A space without any operations
CO2	K1	3.	A continuous linear operator that is surjective between Banach spaces is also names as an a) open map b) closed map c) continuous d) convergent
CO2	K2	4.	Can you "tell" another name for the Open Mapping Theorem? a) closed graph b) bounded convergence c) spectral d) Banach Theorem
CO3	K1	5.	Identify a closed convex subset C of a Hilbert Space contains a unique vector of ____norm. a) greatest b) summation c) difference d) smallest
CO3	K2	6.	Can you "trace" what is the dot product of orthogonal vectors will be? a) 0 b) 1 c) 4 d) 2
CO4	K1	7.	The adjoint of a matrix, also named as the ____ of a matrix. a) adjoint b) conjugate c) rank d) adjugate
CO4	K2	8.	Can you "trace" the name of an operator when A=A∗? a) unitary b) Self-adjoint c) projections d) normal
CO5	K1	9.	Identify the spectrum of an operator refer to in linear algebra is a) The set of eigenvalues of the operator b) The determinant of the operator c) The trace of the operator d) The inverse of the operator
CO5	K2	10.	Can you "outline" the name of λ If T is an operator on a Hilbert Space H, and a non zero vector x satisfies Tx= λx? a) eigen vector b) eigen space c) eigen value d) vector
Course Outcome	Bloom's K-level	Q. No.	**SECTION – B (5 X 5 = 25 Marks)** Answer ALL Questions choosing either (a) or (b)
CO1	K2	11a.	If a and b greater than equal to zero , p & q are greater than 1 such that (1/p)+(1/q)=1 then show that $a^{\frac{1}{p}} + b^{\frac{1}{q}} \leq \frac{a}{p} + \frac{b}{q}$. **(OR)**
CO1	K2	11b.	Explain that a Vector addition and scalar multiplication is jointly Continuous in $\mathcal{B}$(N).

CO2	K2	12a.	Explain natural imbedding of N in N^{**}. **(OR)**
CO2	K2	12b.	If B and B' are Banach space, T is a linear transformation of B into B' and graph T is closed then demonstrate that T is continuous.
CO3	K3	13a.	Construct the proof of Schwarz inequality . **(OR)**
CO3	K3	13b.	Prove that the inner product in a Hilbert space is jointly continuous.
CO4	K3	14a.	If A_1, A_2 are self adjoint operator on H, then illustrate that their product A_1A_2 is self adjoint if and only if $A_1A_2 = A_2 A_1$. **(OR)**
CO4	K3	14b.	Determine that an operator T on H is self adjoint if (Tx, x) is real for all x.
CO5	K4	15a.	Illustrate that i) The self adjoint operators in B(H) form a closed real linear subspace of B(H). Also prove that ii) $N_1 + N_2$, $N_1 N_2$ are normal. **(OR)**
CO5	K4	15b.	Prove that I+A is non-singular if A is a positive operator on H.

Course Outcome	Bloom's K-level	Q. No	<u>**SECTION – C**</u> **(5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K4	16a.	Illustrate the Hahn Banach theorem. **(OR)**
CO1	K4	16b.	Illustrate Minkawskis Inequality.
CO2	K5	17a.	If P is a projection on a Banach space B, M and N are its range and null space then prove that M and N are closed linear subspace of $B = M \oplus N$. **(OR)**
CO2	K5	17b.	If B and B' are Banach space, T is a linear transformation of B into B' then prove that T is an open mapping.
CO3	K5	18a.	Prove that a closed convex subset C of a Hilbert space contains a unique vector of smallest norm. **(OR)**
CO3	K5	18b.	If M and N are closed linear subspace of a Hilbert space H such that $M \perp N$. Then prove that the linear space $M + N$ is also closed.
CO4	K5	19a.	If $\{e_i\}$ is an orthonormal set in a Hilbert space H. Then prove that $\sum \|(x, e_i)\|^2 \leq \|x\|^2$ for every vector x in H. **(OR)**
CO4	K5	19b.	If T is an operator on H for which (Tx,x)=0 for all x , then prove that T=0.
CO5	K6	20a.	If $P_1, P_2, P_3, \ldots, P_n$ are the projections on closed linear subspaces $M_1, M_2, \ldots M_n$ of H, then construct that P=$P_1 + P_2 + P_3 + \cdots + P_n$ is also a projection if and only if the P_i's are pairwise orthogonal and in this case, P is the projection on M = $M_1 + M_2 + \ldots + M_n$. **(OR)**
CO5	K6	20b.	If T is normal. Illustrate that x is an eigen vector of T with the eigen value λ iff x is an eigen vector of T^* with the eigen value $\bar{\lambda}$.